

## ELEMENTS OF CONTROL SYSTEMS

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### Summary

This paper presents a perspective of the elements of control systems. Human engineered control systems form part of automation that is characteristic of our society, particularly in the present times. Systems are made as collections of certain individual elements assembled and connected in specific ways to perform the functions for which they are intended. Systems are controlled to meet specified needs and control techniques

enhance their performance as control systems. We understand the behavior of systems by modeling, simulation and analysis. Mathematical models of dynamical systems can be obtained either in the time domain or in the frequency domain. A particular model for a system can be obtained in a chosen form by determining the numerical values of the parameters associated with the model based on input-output data.

This process is known as system identification. Feedback control can be designed for a system with a known model with reference to certain performance criteria such as stability, steady-state accuracy, optimality, disturbance rejection, etc. Controller action can be realized in a computer that works with sampled signals. In the presence of uncertainties and unknown disturbances, stochastic estimation and control techniques are to be applied. When the plant characteristics vary during the period of operation, adaptive control techniques may be used to render the controller adaptive to the changing conditions.

Supported by powerful computational facilities in the control environment, features such as learning and decision-making can be incorporated to render control as intelligent, and control systems can be made fully automatic and autonomous. The history of control dates back to the ancient times, but the beginning of an era of theory and practice of automatic control was made in the 18th century following the inception of the governor as a device for speed control of the steam engines. Major developments took place in the 20th century.

## **1. Introduction**

Systems are sets of components, physical or otherwise, which are connected in such a manner as to form and act as entire units. Control is the effort to make systems act as desired. A process is the action of a system or alternatively, a system in action.

Humans have created control systems as technical innovations to enhance the quality and comfort of their lives. Human engineered control systems are part of automation, which is a feature of our modern life. They are applied in several aspects of our daily life, in heating and air conditioning to control our living environment and in many of our household appliances.

Significantly, they relieve us from the burden of operation of complex systems and processes, and enable us to achieve control with the desired precision. Control systems enable accurate positioning and control of machine tools in metal cutting operations and automate manufacturing processes.

They automatically guide and control space vehicles, aircraft, large sea going vessels, and high-speed ground transportation systems. Modern automation of a plant involves components such as sensors, instruments, computers and the application of techniques of data processing and control. The principles and techniques of automatic control may be applied in a wide variety of systems in order to enhance the quality of their performance.

Control systems are not human inventions; they have naturally evolved in the Earths

living system. The action of automatic control regulates the conditions necessary for life in almost all living things. They possess sensing and controlling systems and counter disturbances. An automatic temperature control system, for example, makes it possible to maintain the temperature of the human body constant at the right value, despite varying ambient conditions.

The human body is a very sophisticated biochemical processing plant in which the consumed food is processed and in the process glands automatically release the required quantities of chemical substances as and when necessary. The stability of the human body and its ability to move as desired are due to some very effective motion control systems. A bird in flight, a fish swimming in water or an animal on the run, all are under the influence of some very efficient control systems that have evolved within that species.

The field of automatic control is very well developed. The techniques established in this field can be applied to the control of a wide range of systems, engineering systems such as machines and complex plants, natural systems such as biological and ecological systems, and non-physical systems such as economic and sociological systems following the understanding of the similarity of the underlying problems.

Understanding a system for its properties is prerequisite to the creation of a control system for it. Before attempting to control a system, it is essential to know how it generally behaves and responds to external stimuli. Such an understanding is possible with the help of a model. The process of developing a model is known as modeling.

Physical systems are usually modeled by applying the phenomenological laws that govern their behavior. For example, mechanical systems are described by Newton's laws, and electrical systems by Ohm's, Faraday's and Lenz's laws. These laws form the basis for the constitutive properties of the elements in a system.

## **2. System Modeling**

Physical systems may be regarded as energy manipulating units, and modeling them is based on the distribution and transfer of energy taking place within them. Energy from certain sources enters a system schematically as shown in Figure 1, and is manipulated within the system by the various components and subsystems in accordance with their inherent properties, and depending on the manner in which they are connected inside the system.

Energy manipulation phenomena are studied in terms of a pair of variables whose product has the sense of power and thereby the meaning of energy. Some elements store energy and some convert it into another form. When an element converts energy into heat, it is termed a dissipator. The assignment of the term 'dissipator' to such elements seems to be prejudiced by their association with heat, a form of energy that is degenerate and vulnerable to loss or dissipation, although the heat generated may indeed be intended for use, say for heating.

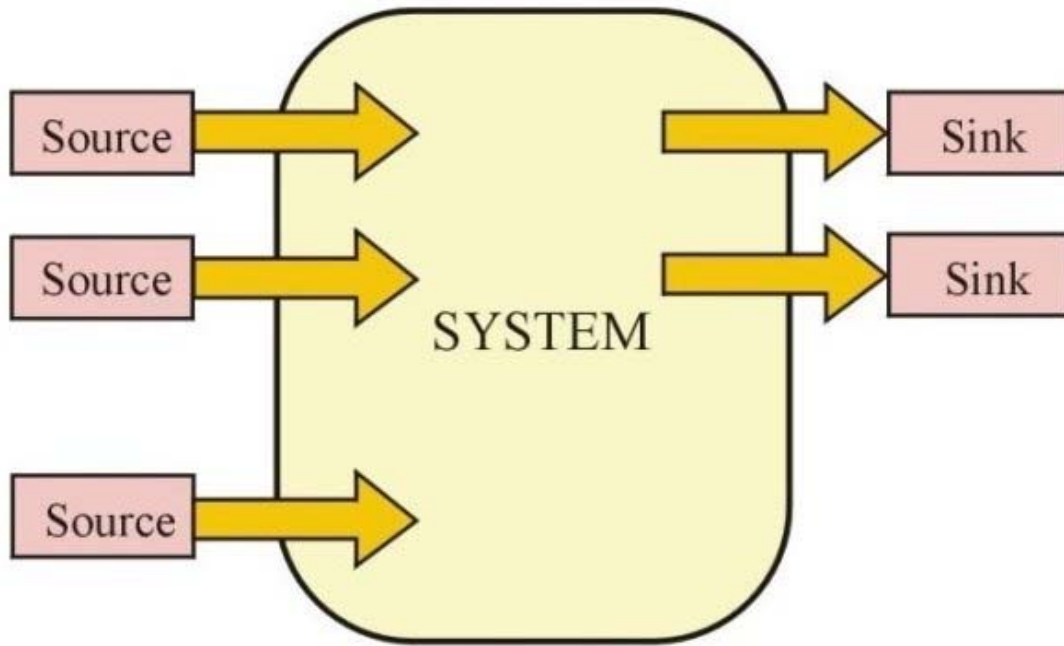


Figure 1. Physical system as an energy manipulator

The energy manipulations in system elements are studied in terms of ‘effort variables’ and ‘flow variables’ whose product corresponds to the ‘rate of energy’ or ‘power’ as indicated in general in Figure 2. For instance, in an electrical system shown in Figure 3, voltage is regarded as an effort variable, and current as the flow variable. Because of the manner in which the effort and flow variables occur, for instance, as voltage across an element and current through it, they are also termed as ‘across’ and ‘through’ variables respectively.

The elements within a given system may have the property to store or dissipate energy. Energy stores are classified as effort stores and flow stores. For example, in electrical systems, inductors accumulate the effort variable (voltage) and capacitors accumulate the flow variable (electric current). Resistors convert electrical energy into heat and are termed as dissipators.

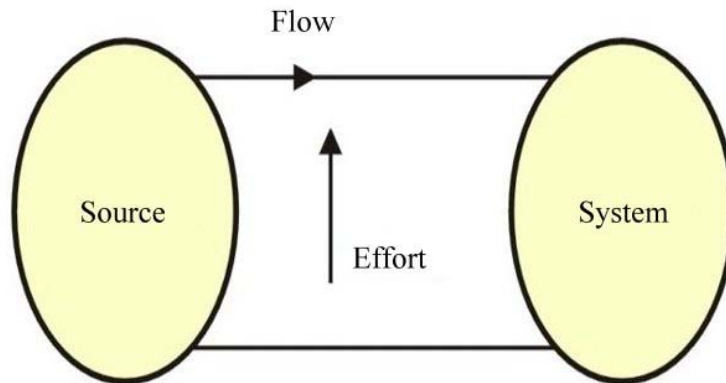


Figure 2. Effort and flow variables

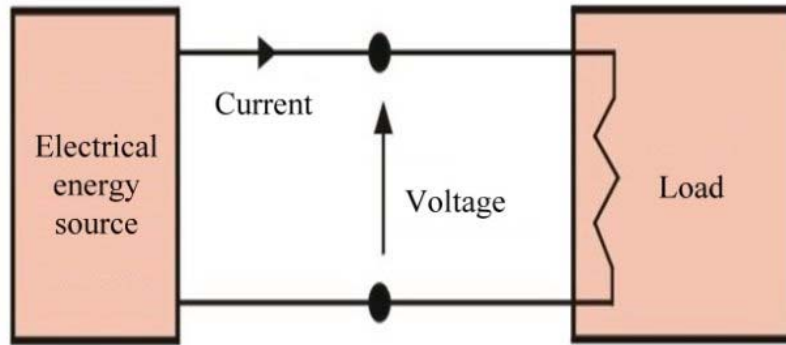


Figure 3. A simple electrical system

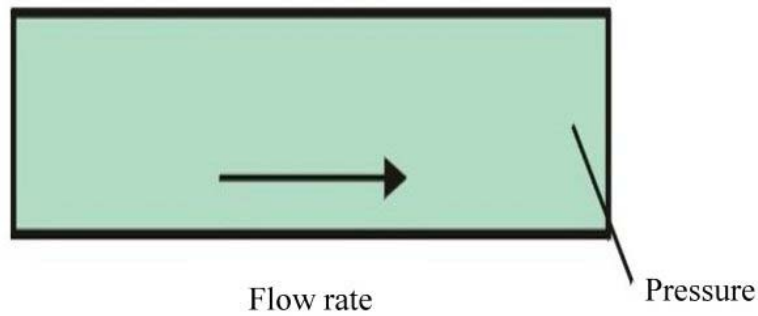


Figure 4. A simple fluid system

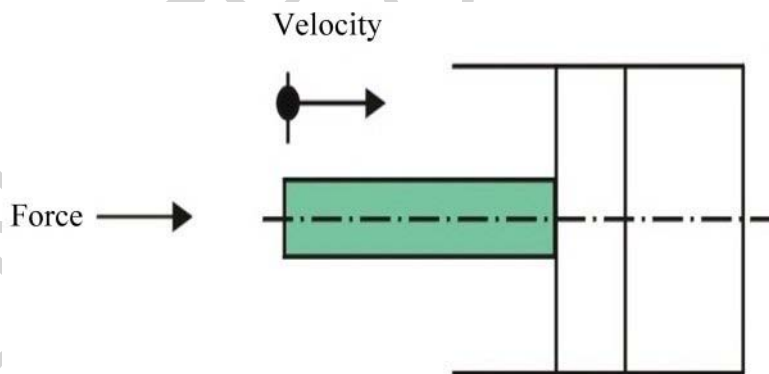


Figure 5. A simple mechanical system

It is the presence of stores that renders a system 'dynamic'. Figures 4 and 5 show the representations in fluid and mechanical systems respectively.

Mathematical modeling of a system is the process of obtaining a mathematical description that adequately describes the aspects of its behavior, which are of interest in the context of a study. Modeling is by itself a well-developed field, and there are some general approaches that are applicable to a wide variety of systems. The following are some important approaches to physical system modeling:

- Network methods
- Variational methods
- Bond graph methods

The network methods of system modeling are based on the generalization of the methods of electrical network theory. First, all the elements in the system are described (modeled) by their constitutive properties in terms of storage, dissipation, and conversion, by applying the physical laws governing their behavior. Next, generalized Kirchhoff’s laws are applied to take into account the connections among the elements in the system. These give rise to the so-called continuity and compatibility conditions, which constrain the effort and flow variables in accordance with the system configuration. As a result of these constraints, the effort and flow variables of the individual elements in a system cannot all be assigned independent labels.

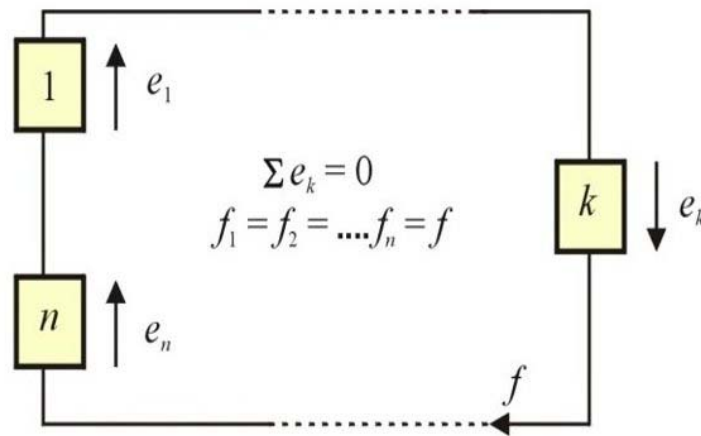


Figure 6. Compatibility constraint on effort variables

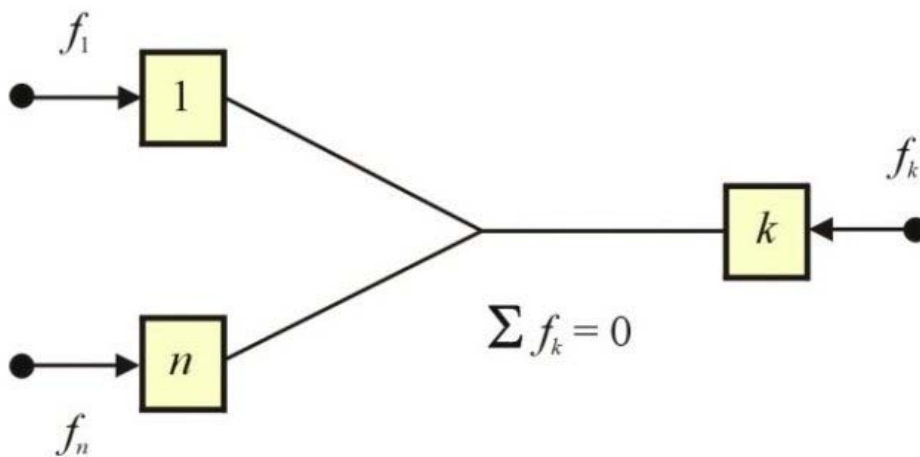


Figure 7. Continuity constraint on flow variables

The variables are bound by the structural configuration of the system or in other words, the manner in which the individual elements are connected in the system. Figure 6 shows how the effort variables in a closed-loop are constrained, and Figure 7 shows

how the flow variables are constrained. The effort variables that represent a loop in the system of Figure 6 are such that their algebraic sum is zero. Likewise, the algebraic sum of the flow variables at a junction is zero. This condition is termed the continuity constraint, because this implies continuity, that is, the inflows and the outflows must be equal at a junction.

Graph theoretic methods may be applied as general tools to apply the interconnectivity constraints. These constraints will eliminate the redundancy in the labels chosen to describe the variables. For example, in the loop of Figure 6, only one flow variable is to be defined, and it applies to all the components by virtue of the series connection. Furthermore, it is enough if all but one of the effort variables in the loop are labeled. The unlabeled variable is naturally determined by the negative sum of these  $n-1$  variables. Thus, application of the interconnectivity constraints brings down the multitude of the system variables to the appropriate number, and to mutual relationships. The resulting equations are then arranged in the desired form to represent the system model.

The variational methods of Lagrange and Hamilton avoid explicit formulation of both sets of interconnectivity constraints. Only one set needs to be directly known, and the other is complementary and implicit in these methods. Complex couplings of different energy handling media are particularly susceptible to the variational approach. In this approach, infinitesimal alterations in certain key system effort or flow accumulation variables, without transgressing the related compatibility or continuity constraints, are considered as admissible variations. A scalar function known as the variational indicator has to be zero in a natural configuration. In this approach, variational calculus, Hamilton's principle and Lagrange's equation are applied. Lagrange's equations, which are in terms of certain energy functions, give rise directly to the differential equations governing the system. This approach is applicable to composite systems containing elements and subsystems belonging to different worlds - electrical, mechanical, etc. Bond graph methods represent the energetic interactions between systems and their components by single lines termed as energy bonds. Bond graph representation is alternative to the network convention and is more compact and orderly than the equivalent system graph. It also allows multiport elements to be modeled explicitly and neatly.

Physical system modeling on the basis of the above approaches can be computer aided and software packages are available for this purpose..

### **3. Mathematical Models of Dynamical Systems**

Mathematical models may be in the form of differential, algebraic or logical equations, depending on the nature of the system (see *General Models of Dynamic Systems*). They are useful in providing an understanding of the input-output behavior and stability studies. They are helpful in the analysis or synthesis of control systems as well as in simulation studies with the help of analog, digital or hybrid computers. The mathematical equations are 'solved' in devices, computational or otherwise, to display the system behavior. Through simulation, we gain an understanding of the performance of a system under different situations, without the need to run the actual system.

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## Biographical Sketch

**Ganti Prasada Rao** was born in Seethanagaram, Andhra Pradesh, India, on August 25, 1942. He studied at the College of Engineering, Kakinada and received the B.E. degree in Electrical Engineering from Andhra University, Waltair, India in 1963, with first class and high honours. He received the M.Tech. (Control Systems Engineering) and Ph.D. degrees in Electrical Engineering in 1965 and 1970 respectively, both from the Indian Institute of Technology (IIT), Kharagpur, India. From July 1969 to October 1971, he was with the Department of Electrical Engineering, PSG College of Technology, Coimbatore, India as an Assistant Professor. In October 1971, he joined the Department of Electrical Engineering, IIT Kharagpur as an Assistant Professor and was a Professor there from May 1978 to June 1997. From May 1978 to August 1980, he was the Chairman of the Curriculum Development Cell (Electrical Engineering) established by the Government of India at IIT Kharagpur. From October 1975 to July 1976, he was with the Control Systems Centre, University of Manchester Institute of Science and Technology (UMIST), Manchester, England, as a Commonwealth Postdoctoral Research Fellow. During October 1981- November 1983, May-June 1985 and May-June 1991, he visited the Lehrstuhl fuer Elektrische Steuerung und Regelung, Ruhr-Universitaet Bochum, Germany as a Research Fellow of the Alexander von Humboldt Foundation. Since June 1992 he is on a visit to Abu Dhabi as Scientific Advisor to the Directorate of Power and Desalination Plants, Water and Electricity Department, Government of Abu Dhabi and the International Foundation for Water Science and Technology where he worked in the field of desalination plant control. He is presently a member of the UNESCO-EOLSS Joint Committee.

He has authored/coauthored four books: *Piecewise Constant Orthogonal Functions And Their Applications to Systems and Control*, *Identification of Continuous Dynamical Systems- The Poisson Moment Functional (PMF) Approach* (with D.C.Saha), *General Hybrid Orthogonal Functions and their Applications in Systems and Control* (with A. Patra) all the three published by Springer in 1983, 1983 and 1996 respectively, and *Identification of Continuous Systems* (with H.Unbehauen) Published by North Holland in 1987. He is Co-Editor (with N.K.Sinha) of *Identification of Continuous Systems - Methodology and Computer Implementation*, Kluwer, 1991. He has co-authored (with A.Patra): *General Hybrid Orthogonal Functions and Their Applications in Systems and Control*, LNCIS-213, Springer, 1996. He has authored/coauthored over 150 research papers. He is on the Editorial Boards of *International Journal of Modeling and Simulation*, *Control Theory and Advanced Technology (C-TAT)*, *Systems Science (Poland)*, *Systems Analysis Modeling and Simulation (SAMS)* and *The Students' Journal of IETE(India)*. He was Guest Editor of two Special Issues: one of *C-TAT on Identification and Adaptive Control - Continuous Time Approaches*, Vol.9, NO.1, March 1993, and *The Students' Journal of IETE on Control*, Vols. I&II, 1992-93. He is on the Honorary Editorial Advisory Board of *The American Biographical Research Institute*. He organized several invited sessions in IFAC Symposia on Identification and System Parameter Estimation, 1988, 1991, 1994 and World Congress 1993. He was a member of the IFAC Technical Committee on Modeling, Identification and Signal Processing in 1996. He was Chairman of the Technical Committee of the 1989 National Systems Conference in India. He is co-editor (with A. Sydow) of the book series “*Numerical Insights Series*” published by Gordon and Breach. He is a member of the Advisory Board of the Internal Study Group on Water and Energy Systems (ISGWES). Over the last several years, he has devoted himself to the development, from concept to completion, of the *Encyclopedia of Desalination and Water Resources* (DESWARE-online) and *Encyclopedia of Life support Systems* (EOLSS), two major publications of EOLSS Publishers, Oxford, UK.

He has received several academic awards including the IIT Kharagpur Silver Jubilee Research Award 1985, The Systems Society of India Award 1989, International Desalination Association Best Paper Award 1995 and Honorary Professorship of the East China University of Science and Technology, Shanghai. The International Foundation for Water Science and Technology has established the ‘Systems and Information Laboratory’ in the Electrical Engineering Department at the Indian Institute of Technology, Kharagpur, in his honor. He is listed in several biographic publications. Professor Rao is a Life Fellow of The Institution of Engineers (India), Fellow of The Institution of Electronics and Telecommunication Engineers (India), Fellow of IEEE (USA) and a Fellow of the Indian National Academy of Engineering.